CHAPTER -13

SURFACE AREAS AND VOLUMES

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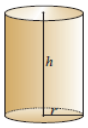
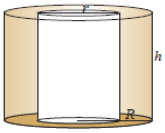
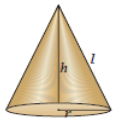
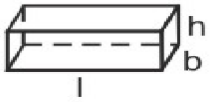
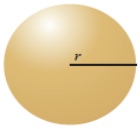
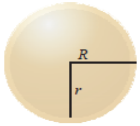
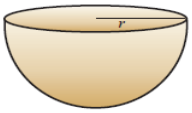
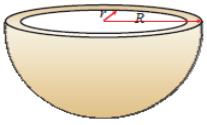
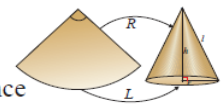
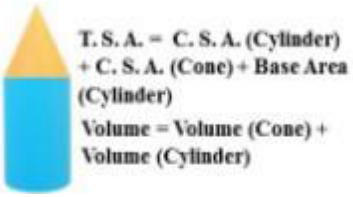
Sl. No	Name	Figure	Lateral or Curved Surface Area (sq.units)	Total Surface Area (sq.units)	Volume (cu.units)
1	**Right circular cylinder**		$2\pi rh$	$2\pi r(h+r)$	$\pi r^2 h$
2	**Right circular hollow cylinder**		$2\pi h(R+r)$	$2\pi(R+r)(R-r+h)$	$\pi R^2 h - \pi r^2 h$ $\pi h(R^2 - r^2)$ $\pi h(R+r)(R-r)$
3	**Right circular cone**		πrl	$\pi r(l+r)$	$\frac{1}{3}\pi r^2 h$
4	Cuboid		2h(l+ b)	*2(lb + bh + hl)*	*lbh*
5	**Sphere**		$4\pi r^2$	- - -	$\frac{4}{3}\pi r^3$
6	**Hollow sphere**		- - -	- - -	$\frac{4}{3}\pi(R^3 - r^3)$
7	**Hemisphere**		$2\pi r^2$	$3\pi r^2$	$\frac{2}{3}\pi r^3$
8	**Hollow Hemisphere**		$2\pi(R^2 + r^2)$	$2\pi(R^2 + r^2)+\pi(R^2 - r^2)$	$\frac{2}{3}\pi(R^3 - r^3)$
9	**Cone**  CSA of a cone = Area of the sector $\pi rl = \frac{\theta}{360} \times \pi r^2$ Length of the sector = Base circumference of the cone			$l = \sqrt{h^2 + r^2}$ $h = \sqrt{l^2 - r^2}$ $r = \sqrt{l^2 - h^2}$ 	10. Volume of water flows out through a pipe = {Cross section area × Speed × Time } 11. No. of new solids obtained by recasting = $\frac{\text{Volume of the solid which is melted}}{\text{volume of one solid which is made}}$	
12	Conversions	1 m^3 = 1000 litres , 1 $d.m^3$ = 1 litre , 1000 cm^3 = 1 litre , 1000 litres = 1 *kl*			

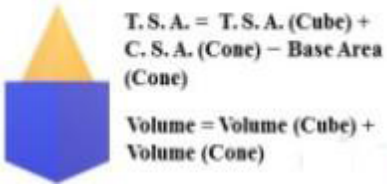
SURFACE AREAS AND VOLUMES OF COMBINATIONS OF SOLIDS

Surface areas and volumes of combinations of any two of the following: cubes, cuboids, spheres, hemispheres and right circular cylinders/cones



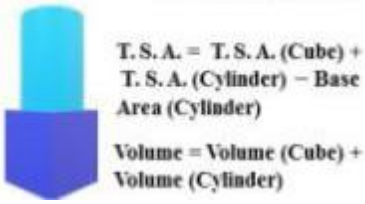
T. S. A. = C. S. A. (Cylinder) + C. S. A. (Cone) + Base Area (Cylinder)

Volume = Volume (Cone) + Volume (Cylinder)



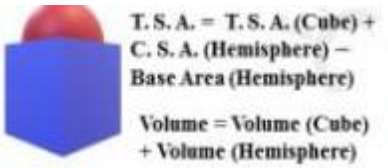
T. S. A. = T. S. A. (Cube) + C. S. A. (Cone) − Base Area (Cone)

Volume = Volume (Cube) + Volume (Cone)



T. S. A. = T. S. A. (Cube) + T. S. A. (Cylinder) − Base Area (Cylinder)

Volume = Volume (Cube) + Volume (Cylinder)



T. S. A. = T. S. A. (Cube) + C. S. A. (Hemisphere) − Base Area (Hemisphere)

Volume = Volume (Cube) + Volume (Hemisphere)

Some solved examples

1. (*) From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2.

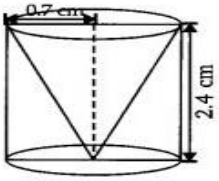
Solution:

Radius of cylinder, $r = \frac{1.4}{2} = 0.7$ cm

Height of cylinder, $h = 2.4$ cm

Radius of cone, $r = 0.7$ cm

Height of cone, $h = 2.4$ cm



Total surface area of the remaining solid

= Outer curved surface area of cylinder
+ Area of bottom of cylinder
+ Inner curved surface area of conical cavity

$= 2\pi rh + \pi r^2 + \pi rl$

$= \pi r(2h + r + l)$

$= \frac{22}{7} \times 0.7\left[2 \times 2.4 + 0.7 + \sqrt{(0.7)^2 + (2.4)^2}\right]$

$= 22 \times 0.1\ [4.8 + 0.7 + 2.5]$

$= 22 \times 0.1 \times 8 = 17.6\ cm^2 \approx \mathbf{18\ cm^2}$.

2.(***) A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.

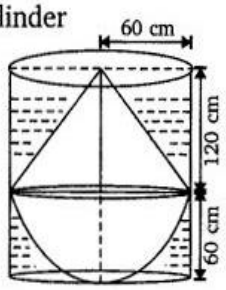
Radius of cylinder, R = 60 cm

Height of cylinder, H = 180 cm

∴ Volume of water in the cylinder

$= \pi R^2 H = \pi(60)^2 \times 180$

$= (\pi \times 3600 \times 180)\text{cm}^3$



Volume of water flows out

= Volume of conical part

+ Volume of hemispherical part

$= \left[\frac{1}{3}\pi(60)^2 \times 120 + \frac{2}{3}\pi(60)^3\right]\text{cm}^3$

$= \left[\frac{4}{3}\pi(60)^3\right]\text{cm}^3$

Volume of water left in the cylinder

$= \left[\pi \times 3600 \times 180 - \frac{4}{3}\pi \times 3600 \times 60\right]\text{cm}^3$

$= \left[3600 \times 60\pi\left(3 - \frac{4}{3}\right)\right]\text{cm}^3$

$= \left[3600 \times 60 \times \frac{22}{7} \times \frac{5}{3}\right]\text{cm}^3$

$= 1131428.57\ \text{cm}^3$

$= \frac{1131428.57}{1000000}\ \text{m}^3 = \mathbf{1.131\ m^3}$ (approx).

MULTIPLE CHOICE QUESTIONS (MCQs)

Q1. In a right angled triangle the sides including the right angles are 3 cm and 4 cm. If the triangle is rotated about 4cm, then the volume of the solid thus generated is

a) $48\ \pi cm^3$ b) $12\ \pi cm^3$ c) $36\ \pi cm^3$ d) $16\ \pi cm^3$

Q2. The ratio of the total surface area of a solid hemisphere to the square of its radius is

a) 2 : 1 b) 3 π: 1 c) 4 π : 1 d) 1: 4 π

Q3. The radius of a wheel is 0.25 m. The number of revolutions it will make to travel a distance of 11 km is

a) 6500 b) 600 c) 7000 d) 7500

Q4. If the volume of a cube is 1331 cm^3, then the length of its edge is

a) 11 cm b) 14 cm c)13 cm d) 12 cm

Q5. If the volume and the surface area of a sphere are numerically equal, then the radius of the sphere is

a) 2 units b) 1 unit c) 3 units d) 4 units

Q6. A cylinder and a cone are of same base radius and of same height. The ratio of the volumes of cylinder to that of the cone is

a) 1:3 b) 2:1 c) 3:1 d) 1:2

Q7. A cylinder, a cone and a hemisphere are of same base and have the same height. The ratio of their volumes is

a) 3:1:2 b) 1:2:3 c) 2:3:1 d) 1:1:3

Q8. If radius of a sphere is $\frac{2d}{3}$ then its volume is

a) $\frac{32}{81}\pi d^3$ b) $\frac{23}{4}\pi d^3$ c) $\frac{32}{3}\pi d^3$ d) $\frac{34}{3}\pi d^3$

Q9. A surahi is the combination of

a) a sphere and a cylinder

b) a hemisphere and a cylinder

c) two hemispheres

d) a cylinder and a cone

Q10. If the diameter of the sphere is doubled, the surface area of the resultant becomes x times that of the original one. Then the value of x is

a) 2 b) 3 c) 4 d) 5

Q11. From a solid circular cylinder with height 10 cm and radius of the base 6 cm, a right circular cone of the same base and same height is removed, then the volume of the remaining solid is

a) $280\ \pi cm^3$ b) $330\ \pi cm^3$ c) $240\ \pi cm^3$ d) $440\ \pi cm^3$

Q12. The edge of a cube whose volume is equal to that of a cuboid of dimensions 8 cm x 4 cm x 2 cm is

a) 6 cm b) 4 cm c) 2 cm d) 6 cm

Q13. The sum of the length , breadth and height of a cuboid is $6\sqrt{3}$ cm and the length of its diagonal is $2\sqrt{3}$ cm . The total surface area of the cuboid is

a) 48 cm^2 b) 72 cm^2 c) 96 cm^2 d) 108 cm^2

Q14. The radius of a sphere is r cm. The sphere is divided into two equal parts. The whole surface areaof two parts will be:

a) $8\pi r^2$ b) $6\pi r^2$ c) $4\pi r^2$ d) $3\pi r^2$

Q15.The radius of a wire is decreased to one third. If the volume remains the same, the length will become

a) 3 times b) 6 times c) 9 times d) 27 times

Q16. The ratio of the volumes of two spheres is 8 : 27 . If r and R are the radii of spheres respectively, then (R – r) : r is :

a) 1 : 2 b) 1 : 3 c) 2 : 3 d) 4 : 9

Q17. The surface area of the two spheres are in the ratio 1 : 2. The ratio of their volumes is :

a) $\sqrt{2}$: 1 b) 1 : $2\sqrt{2}$ c) 1 : 8 d) 1 : 4

Q18. If the areas of three adjacent faces of a cuboid are X, Y and Z respectively, then the volume of cuboid is :

a) XYZ b) 2XYZ c) $\sqrt{XYZ}$ d) $\sqrt{2XYZ}$

Q19. Assertion: A cylinder and right circular cone are having the same base and same height the volume of cylinder is three times the volume of cone

Reason: If the radius of cylinder is doubled and height is halved the volume will be doubled

a) Both Assertion and reason are correct and reason is correct explanation for Assertion

b) Both Assertion and reason are correct but reason is not correct explanation for Assertion

c) Assertion is correct but reason is false

d) Both Assertions and reason are false

Q20. Assertion: Volume of cuboid is defined as the amount of space occupied by the walls of cuboid in three dimensional space

Reason: Volume of cuboid is the product of length ,width ,height

a) Both Assertion and reason are correct and reason is correct explanation for Assertion

b) Both Assertion and reason are correct but reason is not correct explanation for Assertion

c) Assertion is correct but reason is false

d) Both Assertions and reason are false

Q21.Assertion: The lateral surface area of a right cone is 62.82 cm^2, if the radius is 4 cm and the slant height is 5 cm.

Reason: Lateral surface area of cone = πrl

a) Both Assertion and reason are correct and reason is correct explanation for Assertion

b) Both Assertion and reason are correct but reason is not correct explanation for Assertion

c) Assertion is correct but reason is false

d) Both Assertions and reason are false

Q22. **Assertion: Savitri had to make a model of a cylindrical kaleidoscope for her science project. She wanted to use chart paper to make the curved surface of the kaleidoscope. 550cm^2 would be the area of chart paper required by her, if she wanted to make a kaleidoscope of length 25 cm with a 3.5 cm radius.**

Reason: Area of chart paper required = curved surface area of the kaleidoscope= 2πrh

a) Both Assertion and Reason are correct and reason is correct explanation for Assertion.

b) Both Assertion and Reason are false but reason is not correct explanation for Assertion.

c) Assertion is correct but reason is false.

d) Both Assertion and reason are false.

SHORT ANSWER-I (SA-I)

Q23. Find the volume of the largest right circular cone that can be cut out from a cube of edge 4.2 cm.*

Q24. The radii of two cylinders are in the ratio 3:5 and their heights are in the ratio 2:3 . What is the ratio of their curved surface areas?**

Q25. Two cubes each of volume27cm^3 are joined end to end to form a solid. Find the surface area of the solid. ***

Q26.Two cubes each of side 4cm are joined end to end. Find the volume of the resulting solid.*

Q27. Volume and surface area of a solid hemisphere are numerically equal. What is the diameter of hemisphere? *

Q28. If the total surface area of a solid hemisphere is 462 cm^2, find its radius. *

Q29. How many shots each having diameter 3 cm can be made from a cuboidal lead solid of dimensions 9 cm x 11 cm x 12 cm ? **

Q30. The surface area of a sphere is 616cm^2. Find its radius *

Q31.The base radii of 2 right circular cones of the same height are in the ratio 3:5. Find the ratio of their volumes.**

Q32. From a solid cube of side 7 cm , a conical cavity of height 7 cm and radius 3 cm is hollowed out . Find the volume of the remaining solid.***

Q33. Find the volume of the largest right circular cone that can be cut out of a cube whose edge is 9cm.*

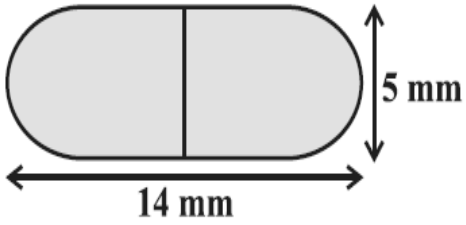
Q34. A toy is in the form of a cone mounted on a hemi-sphere of same radius. The diameter of the base of the conical part is 7cm and the total height of the toy is 14.5cm. Find the volume of the toy. ***

Q35. The rain water from a roof 22m x 20 m drain into a conical vessel having diameter of base as 2m and height 3.5 m. If the vessel is just full, then find the rainfall.*

Q36. The length of a hall is 20m and width is 16m. The sum of the areas of the floor and the flat roof is equal to the sum of the areas of the four walls. Find the height of the hall. **

SHORT ANSWER-II (SA-II)

Q37. A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends. The l length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area.**



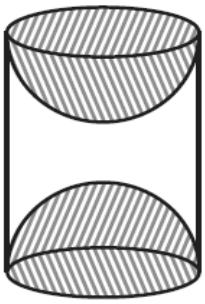
Q38. A circus tent is cylindrical up to a height of 3m and conical above it. If the diameter of the base is 105m and the slant height of the conical part is 53m. Find the total canvas required in making the tent. **

Q39. A bird-bath in a garden is in the shape of a cylinder with a hemi-spherical depression at one end. The height of the hollow cylinder is 1.45m and its radius is 30cm. Find the TSA of the bird-bath.**

Q40. A tent is in the shape of a cylinder of diameter 20m and height 2.5cm, surmounted by a cone of equal base and height 7.5m. Find the capacity of the tent.(take $\pi = 3.14$) ***

Q41. A vessel in the shape of a hollow hemi-sphere mounted by a hollow cylinder. The diameter of the hemi-sphere is 14cm and the total height of the vessel is 13cm. find the inner surface area of the vessel.**

Q42. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in the figure. If the height of the cylinder is 10 cm and its base is of radius 3.5 cm. Find the total surface of the article. ***



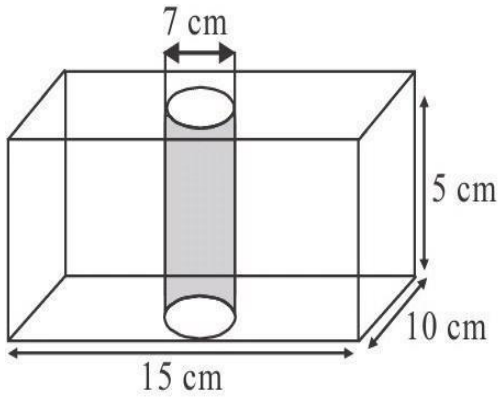
Q43. A waterhouse is used as a granary. It is in the shape of a cuboid surmounted by a half- cylinder. The base of the waterhouse is 6m x 14m and its height is 8m. Find the surface area of non-cuboidal part of the waterhouse.**

Q44. A building is in the form of a cylinder surmounted by a hemispherical vaulted dome and contains $41\frac{19}{21}$ $m^3$of air. If the internal diameter of dome is equal to its height above the floor. Find the height of the building. **

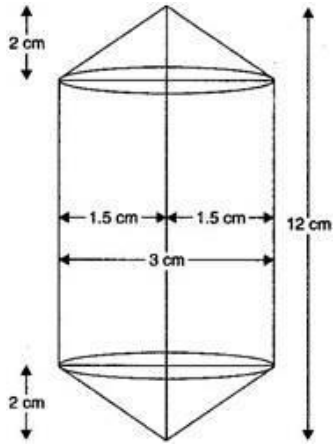
(LONG ANSWER)

Q45.(*) Due to heavy floods in a state, thousands were rendered homeless. 50 schools collectively decided to provide place and the canvas for 1500 tents and share the whole expenditure equally. The lower part of each tent is cylindrical with base radius 2.8 m and height 3.5 m and upper part is conical with the same base radius, but of height 2.1 m . If the canvas used to make the tent costs Rs.120 per m^2, find the amount shared by each school to set up the tents.

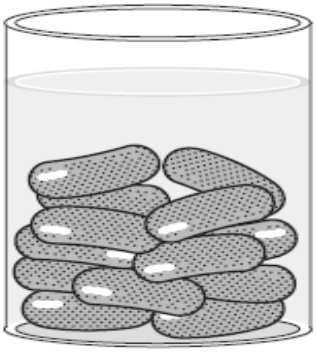
Q46. (**)From a cuboidal solid metallic block of dimensions 15cm x 10cm x 5cm a cylindrical hole of diameter 0.07m is drilled out. Find the surface area of the remaining block. (π= 22/7)



Q47.(**) Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same).



Q48.(**) A *gulab jamun,* contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 *gulab jamuns,* each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm .

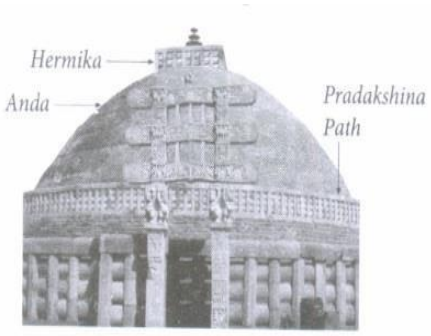


Q49.(*) If a hollow cube of internal edge 22cm is filled with spherical marbles of diameter 0.5 cm and is assumed that $\frac{1}{8}$ space of the cube remains unfilled. Then find the number of marbles that the cube can accommodate.

Q50.(**) From a solid cylinder whose height is 12 cm and diameter is 10 cm, a conical cavity of same diameter is hollowed out. Find the volume and total surface area of the remaining solid.

CASE STUDY BASED QUESTIONS

Q51. Amar is a Class X student. His class teacher Mrs Somya arranged a historical trip to great Stupa of Sanchi. She explained that Stupa of Sanchi is great example of architecture in India. Its base part is cylindrical in shape. The dome of this stupa is hemispherical in shape, known as Anda. It also contains a cubical shape part called Hermika at the top. Path around Anda is known as Pradakshina Path.



Based on the above information, answer the following questions.

(i) Find the lateral surface area of the Hermika, if the side of cubical part is 8 m. (1)

(ii) The diameter and height of the cylindrical base part are respectively 42 m and 12 m. If the volume of each brick used is 0.01 m3, then find the number of bricks used to make the cylindrical base. (2)

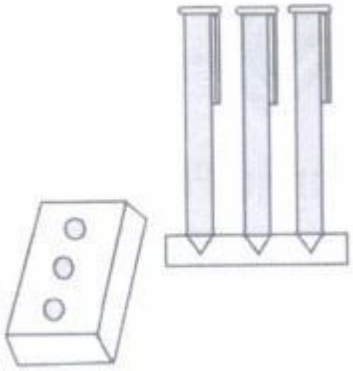
(iii) Find the Curverd surface area of Anda if its radius is 21m. (1)

Q52. A carpenter used to make and sell different kinds of wooden pen stands like rectangular, cuboidal, cylindrical, conical. Aanav went to his shop and asked him to make a pen stand as explained below. Pen stand must be of the cuboidal shape with three conical depressions, which can hold 3 pens. The dimensions of the cuboidal part must be 20 cm x 15 cm x 5 cm and the radius and depth of each conical depression must be 0.6 cm and 2.1cm respectively

i) What will be the volume of cuboidal part? (1)

ii) What is the total volume of conical depression? (1)

iii) Find the total cost of making the pen stand,if the cost of wood used is 0.05 per cm^3. (2)



Q53. Alok and his family went for a vacation to Jaipur. There they had a stay in tent for a night. Alok found that the tent in which they stayed is in the form of a cone surmounted on a cylinder. The total height of the tent is 42 m, diameter of the base is 42m and height of the cylinder is 22 m.

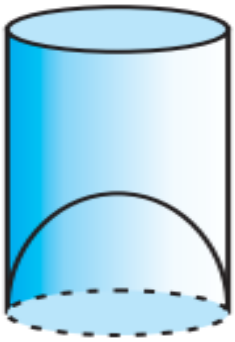
Based on the above information, answer the following questions:

i) How much canvas is needed to make the tent?(1)

ii) If each person needs 126 $m^2$of floor, then how many person can accommodated in the tent?(1)

iii) Find the number of persons that can be accommodated in tent , if each person needs 1892 m^3 of space.(2)

Q54. A juice seller was serving his customers using glasses as shown in Fig. The inner diameter of the cylindrical glass was 5 cm, but the bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass. If the height of a glass was 10 cm, find the apparent capacity of the glass and its actual capacity. (Use π = 3.14.)



i) Find the apparent capacity of the glass. (1)

ii) Find the actual capacity of the glass. (2)

iii) What is the difference between apparent and actual capacity of the glass (1)

Q55. An antique box and its dimensions excluding the stand is given

i) What is the volume of the jewellery box ? (1)

ii) How much brass will be needed to plate the curved surface of the dome as shown in figure? (2)

iii) How many sheets of dimensions 14cm x 30cm x 2cm can be placed in the box? (1)

CHAPTER – 13 ANSWER KEY

Q.NO.	ANSWER	Q.NO.	ANSWER
1	b) 12π cm^3	31	2:5
2	b)3 π:1	32	277 cm^3
3	c) 7000	33	462 cm^3
4	a) 11 cm	34	190.93 cm^3
5	d) 4 units	35	0.83 cm
6	c) 3:1	36	3.5 cm, 7 cm
7	a)3:1:2	37	220 mm^2
8	a)$\frac{32}{81}\pi d^3$	38	28 cm
9	a) A sphere and a cylinder	39	9735 m^2
10	C) 4	40	3.3 m^2
11	c)240 π cm^3	41	1570 m^2
12	C)2 cm	42	374 cm^2
13	C)96 cm^2	43	$\frac{1122}{7}$ m^2
14	b) $6\pi r^2$	44	4m
15	c) 9 times	45	Rs. 3,32,640
16	a) 1:2	46	583 cm^2
17	b)1:2$\sqrt{2}$	47	66 cm^3
18	c)$\sqrt{xyz}$	48	338 cm^3
19	b	49	142296
20	b	50	628.57 cm^3/ 660 cm^2
21	a	51	i)256 m^2 ii) 16,63,200 bricks iii)882π cm^2
22	a	52	i)1500 cm^3 ii)2.376 cm^3 iii)Rs. 748.80
23	19.404^3	53	i)4818 m^2 ii)11 iii)21
24	2:5	54	i)196.25 cm^3 ii)163.54 cm^3 iii)32.71 cm^3
25	90 cm^2	55	i)lbh+$\frac{1}{2}\pi r^2 h$ ii) 660 cm^2 iii)5
26	128 cm^3		
27	9 units		
28	7 cm		
29	84		
30	7cm		

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